Summer Review Packet for Algebra 2

*for students entering Algebra 2 in September*

## Real Numbers and their Properties

### The Real Number System

$\mathbb{R}$: The **Real Numbers** are the Rational Numbers plus the Irrational Numbers.

$\mathbb{Q}$: **Rational Numbers** are all numbers that can be written as fractions. These include any finite decimals and any repeating decimals. Examples are: 0.523, $0.\overline{22}$, ½.

$\mathbb{Z}$: **Integers** are the whole numbers plus the negative natural numbers: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

**Whole Numbers** are the Natural Numbers plus zero: $\{0, 1, 2, \ldots\}$

$\mathbb{N}$: **Natural Numbers** or **Counting Numbers** are the numbers you use when you count: $\{1, 2, 3, 4, \ldots\}$

**Irrational Numbers**: These are Real Numbers that cannot be written as fractions.
Examples are: $\pi$, $\sqrt{2}$, and any decimal that doesn't stop and doesn't have a repeating pattern.

- Natural Numbers, a.k.a. Counting Numbers – the numbers you naturally use to count

$$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$$

- Whole Numbers – the Natural Numbers and zero

$$\{0, 1, 2, 3, \ldots\}$$

- Integers – Whole Numbers and their additive inverses (negatives)

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

- Rational Numbers $\mathbb{Q}$– Ratios (or fractions) of Integers

$$\frac{2}{3} = 0.\bar{6}, \qquad \frac{3}{4} = 0.75, \qquad \frac{-5}{1} = -5$$

- Irrational Numbers – Numbers that, when written as decimals, never terminate nor repeat

$$\pi \approx 3.1415\ldots \qquad \sqrt{2} \approx 1.414213\ldots$$

- Real Numbers $\mathbb{R}$ – All numbers on the number line

  This system contains all of the above number types.

**Properties of Real Numbers**

These are used to solve algebra equations

Let $m$, $n$, and $p$ be real numbers

| Property | Addition | Multiplication |
|---|---|---|
| Closure | $m + n$ is a real number | $m \cdot n$ is a real number |
| Commutative | $m + n = n + m$ | $m \cdot n = n \cdot m$ |
| Associative | $(m + n) + p = m + (n + p)$ | $(m \cdot n) \cdot p = m \cdot (n \cdot p)$ |
| Identity | Zero: $0 + m = m + 0 = m$ | One: $1 \cdot m = m \cdot 1 = m$ |
| Inverse | $m + (-m) = 0$<br>The opposite of $m$ is $-m$ | $m \cdot \frac{1}{m} = 1,\ m \neq 0$<br>The multiplicative inverse is the reciprocal. |
| Distributive | $m \cdot (n + p) = m \cdot n + m \cdot p$ | |

**Exercises A:**

Identify the property being performed

1. $8 \cdot 6 = 6 \cdot 8$
2. $4(6 + 3) = 4 \cdot 6 + 4 \cdot 3$
3. $7(1) = 7$
4. $-4 + 4 = 0$
5. $(9 \cdot 2) \cdot 3 = 9 \cdot (2 \cdot 3)$
6. $(4 + 5) + 3 = 4 + (5 + 3)$

Complete the chart by placing a check in the boxes that apply.

| | Natural Numbers | Whole Numbers | Integers | Rational Numbers | Irrational Numbers | Real Numbers |
|---|---|---|---|---|---|---|
| -2 | | | | | | |
| 5 | | | | | | |
| $\sqrt{2}$ | | | | | | |
| $\frac{8}{9}$ | | | | | | |

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## Algebraic Expressions

The difference between an algebraic expression and an algebraic equation is that an expression does not have an equal sign while and equation does.

| Algebraic Expression | Algebraic Equation |
|---|---|
| $3x + 5$ | $3x + 5 = 9$ |
| Simplify it using the order of operations and by combining like terms<br>P (Parenthesis)<br>E (Exponents)<br>M & D (Multiplication & Division)<br>A & S (Addition & Subtraction) | Solve it by using algebra |

Examples: Evaluate without using a calculator

1. $56 - 12 \div 3 \cdot 2$
   *Solution:* $56 - 4 \cdot 2 = 56 - 8 = 48$

2. $-2(9 - 3^2) + 4^3 \div 2$
   *Solution:* $-2(9 - 9) + 64 \div 2 = -2(0) + 32 = 0 + 32 = 32$

**Exercises B:**

Evaluate for the given value of $x$:

1. $x^3 \div 9 - 2x$ when $x = -3$
2. $\dfrac{(3x^2 - 5x) \div 2}{7x - 10}$ when $x = 5$

Simplify without a calculator:

3. $16 \div (2(3^3 - 11) \div 4) + 5^2$
4. $\dfrac{2(5-7)^3}{\frac{1}{5}} + (15 \div 3 \times 2)$
5. Simplify the expression
   $-3(x^2 + 2x) - 5x(2x - 3)$

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## Solving Linear Equations

Example #1: Solve $-\frac{2}{3}x + 5 = 13$.

*Solution:*

$$-\frac{2}{3}x + 5 = 13$$

$$-\frac{2}{3}x = 8 \qquad \text{Subtract 5 from both sides.}$$

$$x = -12 \qquad \text{Multiply by the reciprocal of } -\frac{2}{3}.$$

$$\checkmark: -\frac{2}{3}(-12) + 5 = 13 \qquad \text{Check using substitution}$$

$$13 = 13 \checkmark$$

Example #2: Solve $6x - 13 = 22 - x$

*Solution:*

$$6x - 13 = 22 - x$$

$$7x - 13 = 22 \qquad \text{Add } x \text{ to both sides.}$$

$$7x = 35 \qquad \text{Add 13 to both sides.}$$

$$x = \frac{35}{7} = 5 \qquad \text{Multiply by the reciprocal of 7 and reduce.}$$

$$\checkmark: 6 \cdot 5 - 13 = 22 - 5 \qquad \text{Check using substitution.}$$

$$17 = 17 \checkmark$$

Example #3: Solve $-2(3x+4)+4x=-3(x+2)$

*Solution:*

$$-2(3x+4)+4x=-3(x+2)$$

$$-6x-8+4x=-3x-6 \qquad \text{Use the distributive property.}$$

$$-2x-8=-3x-6 \qquad \text{Combine like terms.}$$

$$x-8=-6 \qquad \text{Add } 3x \text{ to both sides.}$$

$$x=2 \qquad \text{Add 8 to both sides.}$$

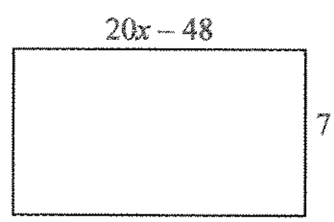
$$\checkmark: -2(3\cdot 2+4)+4\cdot 2=-3(2+2) \qquad \text{Check using substitution.}$$

$$-12=-12$$

**Exercises C:**

Solve the equation and check your solution.

1. $-3x+14=11$
2. $\frac{1}{2}x-8=-3$
3. $4x-12=-3x+9$
4. $\frac{2}{3}x-2=-\frac{3}{2}x-4$
5. $6(-x-5)=-4(x-3)-x$
6. Find the dimensions of the rectangle given that its area is 504 $m^2$.

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## Rewriting Equations and Formulas

To solve for a particular variable means to isolate that variable (get it by itself). You do this using the same solving techniques from the previous section.

Example: Solve $6x-2y=10$ for $y$.

*Solution:*

$$6x-2y=10$$

$$-2y=-6x+10 \qquad \text{Subtract } 6x \text{ from both sides}$$

$$y=-\frac{1}{2}(-6x+10) \qquad \text{Multiply by the reciprocal of } -2.$$

$$y=3x-5 \qquad \text{Distribute and simplify.}$$

Example: Given the equation $2x^2 - 3xy = 9$, find the value of $y$ when $x = -3$.

*Solution #1:*

Solve for $y$ and then substitute.

$$2x^2 - 3xy = 9$$
$$-3xy = -2x^2 + 9$$
$$y = \frac{-2x^2 + 9}{-3x}$$
$$y = \frac{2}{3}x - \frac{3}{x}$$
$$y = \frac{2}{3}(-3) - \frac{3}{-3}$$
$$y = -1$$

*Solution #2:*

Substitute and then solve for $y$.

$$2x^2 - 3xy = 9$$
$$2(-3)^2 - 3(-3)y = 9$$
$$2(9) + 9y = 9$$
$$18 + 9y = 9$$
$$y = -1$$

The area you will use the skill of solving for one variable is when you are using formulas. The following are some formulas your Algebra 2 teacher expects you to know.

- Area of a Triangle: $A = \frac{1}{2}bh$
- Area of a Parallelogram: $A = bh$
- Perimeter of a Rectangle: $P = 2b + 2h$
- Area of a Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$
- Area of a Circle: $A = \pi r^2$
- Equation of a Circle: $(x-h)^2 + (y-k)^2 = r^2$
- Circumference of a Circle: $C = 2\pi r$; $C = \pi d$
- Quadratic Formula: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: Solve for $h$ in the area of a trapezoid formula.

$$A = \frac{1}{2}h(b_1 + b_2)$$
$$2A = h(b_1 + b_2)$$
$$\frac{2A}{b_1 + b_2} = h$$

**Exercises D:**

1. Solve for $y$: $2x - 3y = 24$ when $x = 6$
2. Solve for $y$: $2xy - 3y = -91$ when $x = -5$
3. Solve for the diameter in the circumference of a circle formula.
4. Write an equation to represent the situation. Then solve.

   You are taking horseback riding lessons. The cost of the introductory lesson is two-thirds the cost of each additional lesson. If you take a total of six lessons and spend a total of \$340, how much was the introductory lesson?
5. Write an equation to represent the situation. Then solve.

   You purchase three bags of fertilizer for your vegetable garden. Each bad contains fifty cubic feet of fertilizer. If the fertilizer is spread a single foot thick and the length of the garden is five more than the width, what are the dimensions of the garden?

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## Solving Linear Inequalities

Linear Inequalities in one variable are graphed on a number line. An open circle ○ or parenthesis ( ) are used to represent the less than < and greater than > symbols. A closed circle ● or brackets [ ] are used to represent the less than or equal ≤ and greater than or equal ≥ symbols. You then shade the number line to the left or the right.

A compound inequality is two inequalities joined by an "and" or an "or."

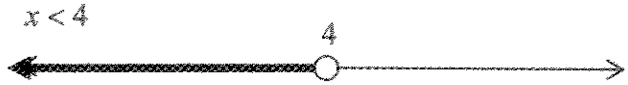
You solve an inequality just like you solve an equation, except for when you multiply or divide by a negative number. Then you have to switch the direction of the inequality symbol.

Example #1: Solve and graph $3x - 4 < 8$.

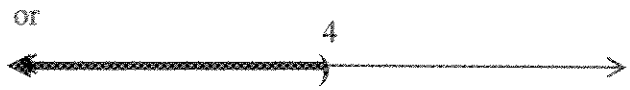
*Solution:* $3x - 4 < 8$

$3x < 12$

$x < 4$




or



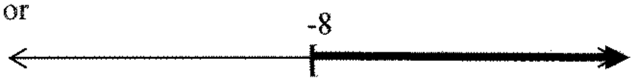
Example #2: Solve and graph $-2x+3\ge -3x-5$.

*Solution:* $-2x+3\ge -3x-5$

$x+3\ge -5$

$x\ge -8$


or




Example #3: Solve and graph $-4\le -x+6<10$.

*Solution:* $-4\le -x+6<10$

$-10\le -x<4$

$10\ge x>4$

$4<x\le 10$


Example #4: Solve and graph $x-4>-2$ *or* $-x-2>2$.

*Solution:* $x-4>-2$ *or* $-x-2>2$

$x>2$ *or* $-x>4$

$x>2$ *or* $x<-4$

$x<-4$ *or* $x>2$


**Exercises E:**

Solve and graph:

1. $-3x+9<12$
2. $7x-10\ge 11$
3. $-\frac{2}{3}x-4\le 8$
4. $-4<2x\le 8$
5. $-3x+5<-10$ *or* $4x-1\le 3$

## Absolute Value Equations and Inequalities

The first step in solving any of these problems algebraically is to remove the absolute values:

| $\lvert ax+b\rvert=c$ | $\lvert ax+b\rvert<c$ | $\lvert ax+b\rvert>c$ |
|---|---|---|
| $ax+b=-c$ or $ax+b=c$ | $-c<ax+b<c$ | $ax+b<-c$ or $ax+b>c$ |
| Example: $\lvert 2x-5\rvert=11$ | Example: $\lvert -x+4\rvert\le 6$ | Example: $\lvert 4x-6\rvert\ge 8$ |
| $2x-5=11$ $\quad 2x-5=-11$ | $-6\le -x+4\le 6$ | $4x-6\le -8$ $\quad 4x-6\ge 8$ |
| $2x=16$ or $2x=-6$ | $-10\le -x\le 2$ | $4x\le -2$ or $4x\ge 14$ |
| $x=8$ $\quad x=-3$ | $10\ge x\ge -2$ | $x\le -\frac{1}{2}$ $\quad x\ge \frac{7}{2}$ |

**Exercises F:**

1. Solve $|4x-6| \le 10$.
2. Solve $\frac{1}{2}|5-x| > 4$.
3. Solve $|8x-2| = 14$.
4. Solve $|x-8|+4 < -12$.
5. Solve $2|7x+9|-5 = 55$.

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## Slope

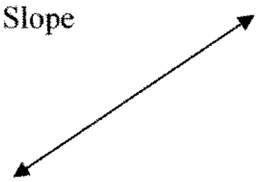
Given two points $(x_1, y_1)$ and $(x_2, y_2)$, the slope of the line through them is $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$.

Example: Find the slope of the line passing through the points (4, -2) and (0, 3).

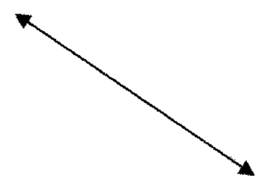
*Solution:* $m = \frac{3-(-2)}{0-4} = -\frac{5}{4}$.

Slope is classified as:

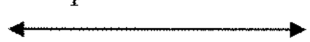
Positive Slope



Negative Slope



Zero Slope



Undefined Slope

*Parallel Lines* have the same slope.

*Perpendicular lines* have opposite reciprocal slopes.

**Exercises G:**

For questions 1-4, find the slope of the line passing through the given points. Then tell whether the line has positive slope, negative slope, is horizontal (zero slope) or is vertical (undefined slope).

1. (-2, 3) and (4, 5)
2. (4, -5) and (4, 6)
3. (4, 9) and (7, 5)
4. (-3, 6) and (0, 6)

For questions 5 and 6, tell whether $l \parallel m$, $l \perp m$, or neither

5. Line *l*: (-2, 6) and (-2, 4); Line *m*: (6, 8) and (-3, 8)
6. Line *l*: (-2, 4) and (5, 6); Line *m*: (1, -5) and (8,-3)
7. Which line is steeper? Provide numerical proof to support your evidence.
   Line *l*: (2, -4) and (3, 5); Line *m*: (3, -5) and (0, 4)

## Graphing Linear Equations

Slope-Intercept Form of the equation of a line: $y = mx + b$, where $m$ = slope, $b$ = $y$-intercept

To graph: Plot the point (0, $b$). From there, count the rise and run. Connect the dots.

Example: Graph $y = -\frac{2}{3}x + 4$

*Solution:* Plot (0, 4). Then go down 2 and right 3.

Point-Slope form of the equation of a line: $y - y_1 = m(x - x_1)$, where $m$ = slope, $(x_1, y_1)$ is a point on the line.

To graph: Plot the point $(x_1, y_1)$. From there, count the rise and run. Connect the dots.

Horizontal Line: $y = c$, where $c$ is the $y$-intercept

Vertical Line: $x = c$, where $c$ is the $x$-intercept

The intercepts of any graph occur when the graph crosses the $x$-axis or the $y$-axis.

- To find the $x$-intercept, substitute $y = 0$ and solve for $x$.
- To find the $y$-intercept, substitute $x = 0$ and solve for $y$.

## Solutions

Exercises A (page 3)

1. Commutative Property of Multiplication
2. Distributive Property
3. Multiplicative Identity
4. Additive Inverse
5. Associative Property of Multiplication
6. Associative Property of Addition

| | Natural Numbers | Whole Numbers | Integers | Rational Numbers | Irrational Numbers | Real Numbers |
|---|---|---|---|---|---|---|
| -2 | | | ✓ | ✓ | | ✓ |
| 5 | ✓ | ✓ | ✓ | ✓ | | ✓ |
| $\sqrt{2}$ | | | | | ✓ | ✓ |
| $\frac{8}{9}$ | | | | ✓ | | ✓ |

Exercises B (page 4)

1. 3
2. 1
3. 27
4. −70
5. $-13x^2 + 9x$

Exercises C (page 5)

1. $x = 1$
2. $x = 10$
3. $x = 3$
4. $x = -\frac{12}{13}$
5. $x = -42$
6. $x = 6 \therefore$ dimensions are 7 meters by 72 meters

Exercises D (page 7)

1. $y = -4$
2. $y = 7$
3. $d = \frac{C}{\pi}$
4. Let $x$ be the cost of a "regular" horseback riding lesson.
   $\frac{2}{3}x + 5x = 340$
   Price of a regular lesson is \$60, price of an introductory lesson is \$40.
5. Let $x$ be the width of the garden. Then $x + 5$ is the length of the garden.
   $V = x(5+x)1 = 150$, so dimensions are 10 by 15 by 1.

Exercises E (page 8)

1. $x > -1$
2. $x \geq 3$
3. $x \geq -18$
4. $1 < x \leq 7$
5. $x \leq 1$ or $x > 5$

Exercises F (page 9)

1. $-1 \leq x \leq 4$
2. $x < -3$ or $x > 13$
3. $x = 2$ or $x = -\frac{3}{2}$
4. Cannot be solved: absolute values cannot be negative.
5. $x = 3$ or $x = -\frac{40}{7}$

Exercises G (page 9)

1. $\frac{1}{3}$, positive slope
2. Undefined slope, vertical line
3. $-\frac{4}{3}$, negative slope
4. 0, horizontal line
5. $l$: undefined; $m$: zero $\therefore$ perpendicular
6. $l$: $\frac{2}{7}$; $m$: $\frac{2}{7}$ $\therefore$ parallel
7. $l$: 9; $m$: -3 $\therefore$ $l$ is steeper than $m$.